LESSON 6. 2

The Natural Base *e*

Today you will:

- Learn what the Natural Base *e* is, and how it is used in real-world problems.
- Graph natural base exponential functions and interpret them.
- Practice using English to describe math processes and equations

Core Vocabulary:

- Natural base *e*, p. 304

Previous:

- Irrational number
- Properties of exponents
- Percent increase/decrease
- Compound interest

Can you name some of the special numbers we have worked with?

- Pi
  - π
  - ~3.1415962
  - an irrational number
  - the circumference of a circle divided by its diam
- i
  - $\sqrt{-1}$
  - the imaginary number
  - found all over the place for example in electronics

Today we learn a new one!

The Natural Base

- e
- Called the "natural base" (we will see why in a few days)
- ~2.718281828
- Easy to remember: 2.7 1828 1828
- An irrational number
- Turns out to be a very common number! For example, used in finance.
- Also called "Euler's number" after the mathematician who discovered it.
- …by the way, Euler is pronounced "oiler" ☺

Check

You can use a calculator to check the equivalence of numerical expressions involving *e*.

```
e^(3)*e^(6)
      8103.083928
e^(9)
      8103.083928
```

Simplify each expression.

a. $e^3 \bullet e^6$ **b.** $\frac{16e^5}{4e^4}$ **c.** $(3e^{-4x})^2$

SOLUTION

a. $e^3 \bullet e^6$

$= e^9$

b. $\frac{16e^5}{4e^4}$

$= 4e$

c. $(3e^{-4x})^2$

$= 9e^{-8x}$

$= \frac{9}{e^{8x}}$

…these are very simple, just pretend *e* is a variable and simplify!

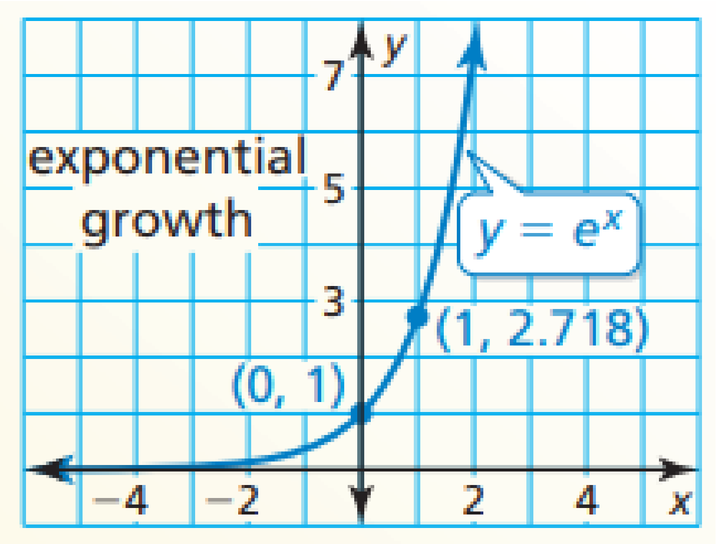
…just remember it is *NOT* a variable, it is 2.718281828…

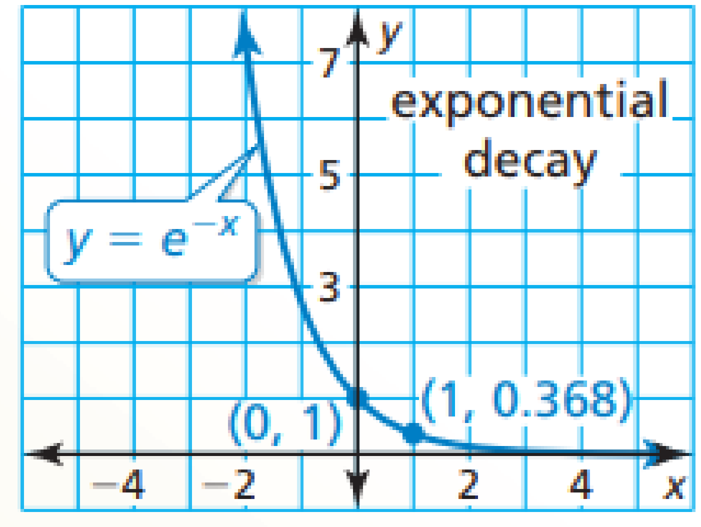
Natural Base Exponential Functions

Exponential functions using the natural base e as the base of the exponent.

Generalize form: $y = ae^{rt}$

- When $a > 0$ and $r > 0$, the function is an exponential growth function.
- When $a > 0$ and $r < 0$, the function is an exponential decay function.





Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

a. $y = 3e^x$

b. $f(x) = e^{-0.5x}$

LOOKING FOR STRUCTURE

You can rewrite natural base exponential functions to find percent rates of change. In Example 2(b),

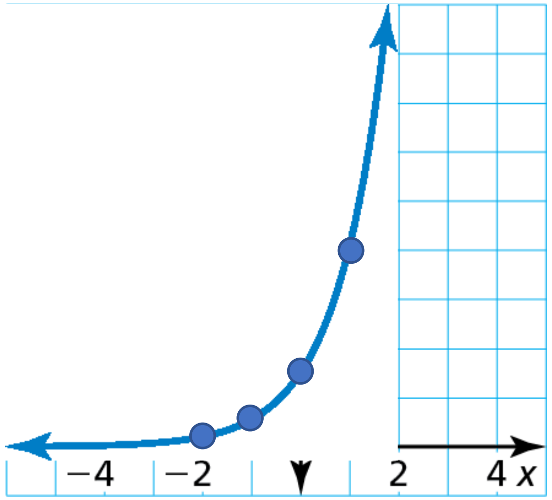
$$f(x) = e^{-0.5x}$$
$$= (e^{-0.5})^x$$
$$\approx (0.6065)^x$$
$$= (1 - 0.3935)^x.$$

So, the percent decrease is about 39.35%.

SOLUTION

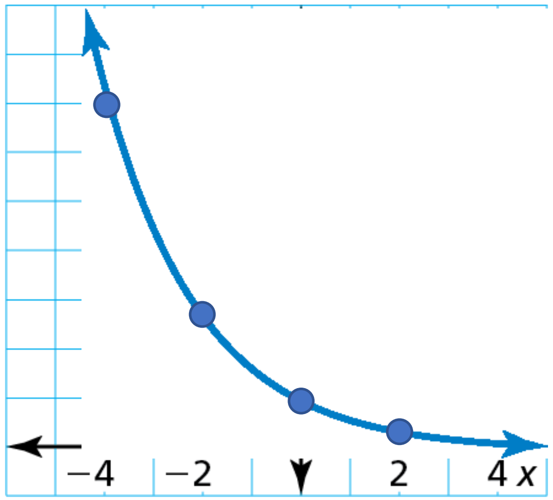
a. Because $a = 3$ is positive and $r = 1$ is positive, the function is an exponential growth function. Use a table to graph the function.

x	-2	-1	0	1
y				



b. Because $a = 1$ is positive and $r = -0.5$ is negative, the function is an exponential decay function. Use a table to graph the function.

x	-4	-2	0	2
y				



Use the properties of exponents to rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then find the percent rate of change. Round to three decimal places.

$$y = e^{-0.75t}$$

SOLUTION

$y = e^{-0.75t}$	$r = -0.75$ (less than 1) so this is decay
$y = (e^{-0.75})^t$	Use power of a power property
$y = (.4723665527)^t$	Simplify
$.4723665527 = 1 - r$	$b \approx .472366527$ and $b = (1 - r)$
$r \approx .528$	Solve for r

The function is $y = (1 - 0.528)^t$ and the rate of decay is .528 or 52.8%.

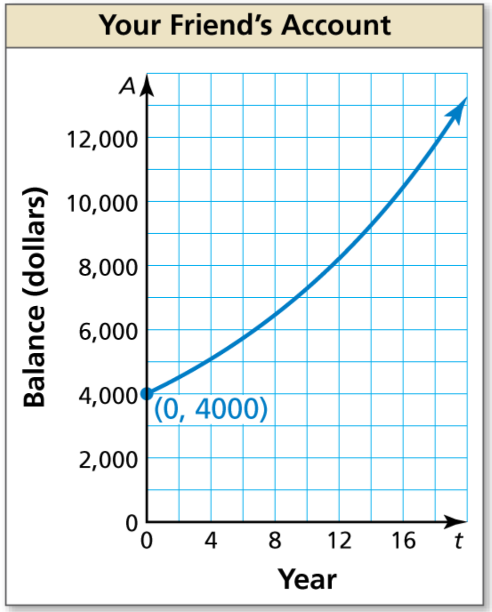
Use the properties of exponents to rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then find the percent rate of change. Round to three decimal places.

$$y = 0.5e^{0.8t}$$

SOLUTION

$y = 0.5e^{0.8t}$	$r = 0.8$ (greater than 1) so this is growth
$y = 0.5(e^{0.8})^t$	Use power of a power property
$y = 0.5(2.2255)^t$	Simplify
$2.2255 = 1 + r$	$b \approx 2.2255$ and $b = (1 - r)$
$r \approx 1.2255$	Solve for r

The function is $y = (1 + 1.226)^t$ and the rate of growth is 1.226 or 122.6%.



You and your friend each have accounts that earn annual interest compounded continuously. The balance A (in dollars) of your account after t years can be modeled by $A = 4500e^{0.04t}$. The graph shows the balance of your friend's account over time. Which account has a greater principal? Which has a greater balance after 10 years?

SOLUTION

1. **Understand the Problem** You are given a graph and an equation that represent account balances. You are asked to identify the account with the greater principal and the account with the greater balance after 10 years.
2. **Make a Plan** Use the equation to find your principal and account balance after 10 years. Then compare these values to the graph of your friend's account.
3. **Solve the Problem** The equation $A = 4500e^{0.04t}$ is of the form $A = Pe^{rt}$, where $P = 4500$. So, your principal is \$4500. Your balance A when $t = 10$ is

$$A = 4500e^{0.04(10)} = \$6713.21.$$

Because the graph passes through (0, 4000), your friend's principal is \$4000. The graph also shows that the balance is about \$7250 when $t = 10$.

So, your account has a greater principal, but your friend's account has a greater balance after 10 years.

4. Look Back Because your friend's account has a lesser principal but a greater balance after 10 years, the average rate of change from $t = 0$ to $t = 10$ should be greater for your friend's account than for your account.

Your account: $\dfrac{A(10) - A(0)}{10 - 0} = \dfrac{6713.21 - 4500}{10} = 221.321$

Your friend's account: $\dfrac{A(10) - A(0)}{10 - 0} \approx \dfrac{7250 - 4000}{10} = 325$

MAKING CONJECTURES

You can also use this reasoning to conclude that your friend's account has a greater annual interest rate than your account.

Homework

Pg 307, #3-35 odd